Optimum Vehicle Acceleration Profile for Minimum Human Injury

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Incidence of human injury during rapid vehicular accelerations may be reduced by employing an acceleration profile that is impulsive and of high amplitude near the beginning and end of the acceleration period and relatively smooth and of low amplitude during the interposed major time segment. This result from optimum control theory, when applied to a validated aircraft ejection seat/human occupant model, reduced by an order of magnitude the injury probability predicted by that model. An idealized acceleration model retains the essential features of the optimum solution and provides rule-of-thumb guidelines for incorporation in system design.

Introduction

IN this paper, an optimum strategy is derived, using optimum control theory, for accelerating a platform bearing human cargo so as to minimize injury. This consideration is particularly important to the designer of systems that protect humans from potentially lethal accelerations, such as stopping a high-speed automobile or ejecting from a disabled aircraft. These two examples present a variety of design considerations, as the former tends to concentrate on minimizing head and abdominal injury and the latter spinal injury. Protection from other types of injury such as limb flail in ejection from aircraft are approached from the viewpoint of restraint design and will not be included in the present discussion.

A basic and well-validated principle of impact protection system design asserts that relatively high-impact acceleration can be tolerated if its time duration is short enough or if its onset is slow enough.[1,2] This principle contrasts the problems of automobile restraint design with those of aircraft ejection seat design, for example. Circumstances surrounding an automobile crash are largely unpredictable, and so vehicle design must provide enough clearance for the occupant to be decelerated by a restraint system that will distribute the inertial forces over a sufficiently long period of time; however, in an aircraft ejection, the constraints are well known to the designer who can concentrate on optimizing the ejection seat rocket (catapult) thrust to provide adequate clearance from the aircraft while minimizing injury potential. As the ejection problem is posed more easily, it is the focus of this paper; however, the concepts presented are more widely applicable.

The criterion of injury probability is incorporated by using the concept of injury index. Several such indices have been developed for injury prediction, and two have been rather well validated epidemiologically; these will be discussed in the next section. Their application requires a dynamic model for the vehicle occupant. Many designers have used linear, lumped parameter models for the occupant and have achieved some degree of success even though recognizing the problem is inherently very nonlinear.[3] Indeed, a second-order, lumped parameter dynamic model for the spine has been very successful in predicting injury probability in aircraft ejection,[4] and that model will be adopted here. The engineer recognizes that a great deal of insight is sometimes gained through the use of mathematically tractable models that only roughly approximate the design problem at hand.

In this paper, optimum control theory is applied to answer the following questions: What is the best (in terms of injury probability) acceleration profile to stop a platform traveling a speed V_0, or equivalently accelerate such a platform from rest to a speed V_0, where either event occurs in time T? The injury probability criterion to be minimized is the Gadd index (discussed in the next section and herein modified to ensure a nonsingular control problem), a functional that has been used in other injury minimization problems that are concerned primarily with head injury; although, its use in other human injury minimization scenarios has been suggested.[5] It turns out that Gadd index minimization also tends to minimize the dynamic response index, which is used frequently in designing ejection seats.[4,6]

The results obtained in this paper are in rough agreement with theoretical predictions of Payne[6] and experimental demonstrations of Hearon et al.[7] Payne[6] showed that, with the cargo modeled as a second-order system, an impulsive acceleration onset followed by a ramped and then constant acceleration tended to reduce the dynamic response index by minimizing cargo displacement overshoot; Hearon et al.[7] showed that a preload in the direction of the anticipated acceleration

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tended to minimize the Gadd index. Both of these excellent papers used ad hoc acceleration profiles; neither sought a globally optimum acceleration profile.

In addition to being consistent with the concepts of dynamic preload, this paper will suggest the use of late, rapid onset dynamic loading, revealing a new consideration for the designer.

Injury Indices

In order to relate biodynamic forces to probability of human injury, two injury indices have become rather widely used: the Gadd index (GI), named for its developer C. W. Gadd,[8] and the dynamic response index (DRI).

Gadd Index

The GI is defined as

$$\mathrm{GI} = \int_0^\infty \left(\frac{a}{g}\right)^n \mathrm{d}t \tag{1}$$

where a is the acceleration experienced by the vehicle occupant, g the acceleration due to gravity, and n an exponent chosen to fit epidemiologic injury data with the goal of rendering GI predictive of injury. This index has been validated primarily through the use of automobile crash data.

Although the GI has been applied primarily to helmet design for minimizing head injury, it has as well been applied to other areas of the body[7]; furthermore, as it is a mathematical functional, it is a natural criterion for minimization exercises using optimum control theory. Deery and McNelis[5] used variational principles to derive an acceleration profile that minimizes this functional for an acceleration over a given distance; however, in contrast to this paper, their work did not use a cargo model.

Dynamic Response Index

Spinal injury occurs when the yield point of a spinal structure is exceeded. The yield point is a displacement sufficiently beyond the elastic limit that structural integrity is breached. As a result of extensive experimentation with human cadaver material, the material strength and dynamic characteristics of the human spine have become fairly well known.[1] This knowledge has led to the use of the DRI in prediction of injury to the spine. The DRI is a function of the maximum displacement experienced by the structure of interest.

The DRI is defined as

$$\mathrm{DRI} = \frac{\delta_{\max}\omega_0^2}{g} \tag{2}$$

where $\delta_{\max}$ is the maximum deflection of the structure, ω_0 its dominant resonance, and g the acceleration due to gravity.

It should be understood that the DRI has been validated as predictive of spinal injury in ejections from aircraft; furthermore, it has proven quite useful in system design for ejection injury mitigation.[9,10] In fact, this method is required to be part of the U.S. Air Force specifications for ejection seats. Using this technique in the design process, aircraft ejection injury rates have been markedly reduced.[4]

In this paper, a new model is introduced that combines the second-order, lumped parameter dynamic model with an injury index criterion in an optimum control design. Because the DRI is proportional to the peak strain in our modeled structure, it is a value at a point in time and is not, therefore, an easily applied criterion in an optimum control model. In contrast, the GI is a functional, the integrand of which includes, through the dynamic model, derivatives of motion as well as displacement; therefore, GI minimization will tend to constrain other aspects of structural motion in addition to displacement. For these reasons, the criterion function chosen for the model presented here is the GI, modified to meet existence and uniqueness requirements of the optimum control model.

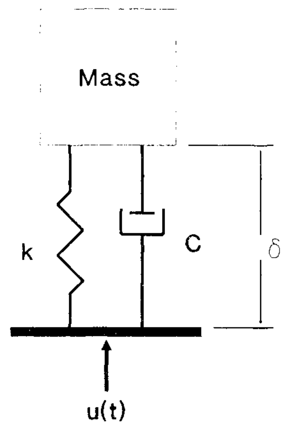

Fig. 1 Second-order, lumped parameter model used for modeling the vehicle cargo: k and C are spring and dashpot constants for these idealized, linear elements, respectively; δ is mass displacement relative to the vehicle and is positive in compression, and $u(t)$ is the acceleration of the vehicle.

Modified Gadd Index

The most frequently applied form of the GI is Eq. (1) with $n = 2.5$, a value that results from fitting automobile crash data; however, values of n from 2 to 3 have been used by others. For mathematical tractability of the control problem, two modifications to the GI are made here: n is fixed at 2.0, and a quadratic function of the applied acceleration, say $u(t)$, is added in the integrand. These modifications guarantee uniqueness of the optimum control solution, thus eliminating one of the more difficult questions surrounding control derivations.

The modified Gadd index (MGI) is defined as

$$\mathrm{MGI} = \int_0^\infty \left[\left(\frac{a}{g}\right)^2 + ru^2\right] \mathrm{d}t \tag{3}$$

where r is a real scalar constant reflecting the weighting given the limitations of the acceleration device (e.g., a rocket motor in an ejection seat). Including such a constraint is practical as the designer must account for such costs.

Dynamic Spinal Injury Model

Figure 1 shows the spinal deflection model used in this paper; the spring and dashpot are assumed ideal. The differential equation describing this system is

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + 2\zeta\omega_n \frac{\mathrm{d}\delta}{\mathrm{d}t} + \omega_n^2\delta = u(t) \tag{4}$$

where δ is deflection, ζ the damping ratio, ω_n the natural frequency of the model, and $u(t)$ the acceleration input.

With reference to Fig. 1, in the case of an ejection seat, $u(t)$ is the applied acceleration pushing the seat from the aircraft, and in the case of an automobile, it is the deceleration of the car.

It is useful to put Eq. (4) in state variable form. Since the terminus in control space will be constrained only to a plane determined by the vehicle's final or initial velocity, an additional state variable will be needed. This will become clear later in the paper. The state variables are defined as

$$x_1 = \int_{t_0}^{t} \delta \, \mathrm{d}t$$

$$x_2 = \delta$$

$$x_3 = \frac{\mathrm{d}\delta}{\mathrm{d}t}$$

Equation (4) in state variable form is, in usual notation,

$$\dot{x} = Ax + Bu \tag{5}$$

where x is the state vector,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2\zeta\omega_n & \omega_n^2 \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

Optimum Control Model

Restating the control objective in terms of the model: find the acceleration profile to stop a platform modeled as in Fig. 1 traveling at velocity V_0, or equivalently accelerate such a platform from rest to a velocity V_0, which minimizes Eq. (3), where either event occurs in time T. To aid the discussion that follows and without loss of generality, only the case with zero initial velocity and V_0 final velocity will be referenced.

Suppose the optimum control is $u^*(t)$ and the optimum trajectory $x^*(t)$, then the problem statement requires

$$\int_0^T u^* \, \mathrm{d}t = V_0 \tag{8}$$

and

$$\{\mathrm{MGI}\}_{\min} = \int_0^\infty \left[\frac{1}{g^2} (2\zeta\omega_n x_3^* + \omega_n^2 x_2^*)^2 + r(u^*)^2 \right] \mathrm{d}t \tag{9}$$

where Eq. (4) has been used in Eq. (3). It can be seen from Eqs. (8) and (4) that the final system state $x(T)$ must lie on the plane

$$x_3(T) + 2\zeta\omega_n x_2(T) + \omega_n^2 x_1(T) = V_0 \tag{10}$$

Hence, Eq. (10) defines the transversality condition for this problem.

In setting up the Hamiltonian, the fact that u is applied only over $[0, T]$ and free motion obtains thereafter must be taken into account. This can be accomplished by expressing the contribution to Eq. (3) for $t > T$ as a closed-form expression in $x(T)$. Using Eqs. (3) and (4) with $u = 0$, $t > T$, and $x|_{t\to\infty} = 0$ gives

$$\mathrm{MGI}|_{t>T} = \frac{1}{2\zeta\omega_n g^2} \{[2\zeta\omega_n x_3(T) + \omega_n^2 x_2(T)]^2 + \omega_n^2 x_3^2(T)\} \tag{11}$$

This expression may be used either to develop a quadratic terminal cost expression or, as its time derivative (with respect to T), to incorporate directly into the criterion functional (3). As it results in somewhat simpler results, the latter approach will be taken; however, both approaches would be entirely equivalent, but only because of the relationships among Eqs. (4), (10), and (11) peculiar to this problem. So, substituting the derivative of Eq. (11) in Eq. (3), the MGI over all motion of the cargo model is, after some simplification,

$$\mathrm{MGI} = \int_0^T \{u[(2\zeta\omega_n + \omega_n/2\zeta)x_3 + \omega_n^2 x_2]/g^2 + ru^2\} \, \mathrm{d}t \tag{12}$$

There is now enough preparation to allow a formal statement of a control problem that will include the problem at hand.

Find a control $u(t) = u^*(t)$ that will take the system,

$$\dot{x} = Ax + Bu \tag{13}$$

from an initial state $x(t_0) = x_0$ to a final state at a specified time T on the smooth surface $g(x)|_{x=x(T)} = 0$, minimizing the functional

$$J = \int_{t_0}^T [\langle u, Qx\rangle + \langle u, Ru\rangle] \, \mathrm{d}t \tag{14}$$

where the matrix R is positive definite. A and B have already been defined; Q and R follow from the previous discussion and will be explicitly defined in the following. This type of control problem is covered in detail in Athans and Falb[11] (Chaps. 5 and 7), and so the derivation of its control law will not be delineated here. Also, because of the particular formulation of this cost functional, Q is not required to be positive semidefinite for u^* to be at least a local minimum; however, for the following control law to hold in general, Q must be positive semidefinite (Ref. 11, Chap. 5, Theorem 5-4).

Control Law 1. For the previously stated problem, an optimum control exists, is unique, and is given by

$$u = -\tfrac{1}{2}R^{-1}[(B^TK + Q)x + B^T\psi\xi] \tag{15}$$

$$\xi = \alpha \left. \frac{\partial g(x)}{\partial x} \right|_{x=x(T)} \tag{16}$$

where Eq. (16) is the transversality condition on the costate vector. The real, symmetric, and positive definite matrix $K(t)$ is the solution of the Riccati-type matrix differential equation,

$$\dot{K} + KC + C^TK - KSK - V = 0 \tag{17}$$

with boundary condition

$$K(T) = 0 \tag{18}$$

The matrix $\psi(t)$ is the solution of the linear matrix differential equation

$$\dot{\psi} + [C^T - KS]\psi = 0 \tag{19}$$

with boundary condition

$$\psi(T) = I \tag{20}$$

where I is the identity matrix, and

$$C = A - \tfrac{1}{2}BR^{-1}Q$$

$$S = \tfrac{1}{2}BR^{-1}B^T$$

$$V = \tfrac{1}{2}Q^TR^{-1}Q$$

Since the solutions to Eqs. (17) and (19) are unique,[11] the optimum control is unique.

For the subject problem as defined by Eqs. (5-8), (10), and (12), the matrices referenced in the optimum control model are

$$Q = \frac{1}{g^2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2\zeta\omega_n & (2\zeta\omega_n + \omega_n/2\zeta) \end{bmatrix} \tag{21}$$

$$S = \frac{1}{2r}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$R = r \cdot I \tag{23}$$

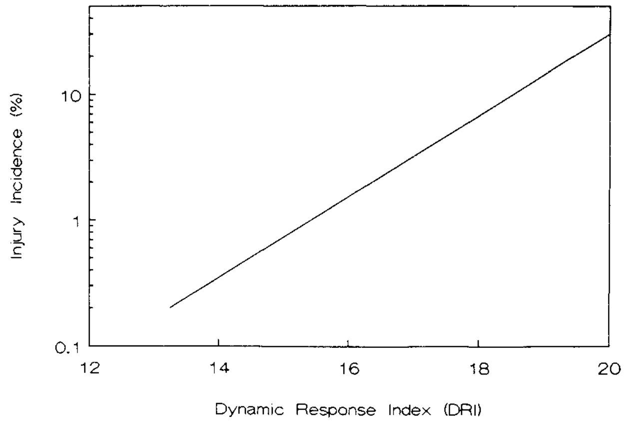

Fig. 2 Spinal injury incidence as a function of the DRI.

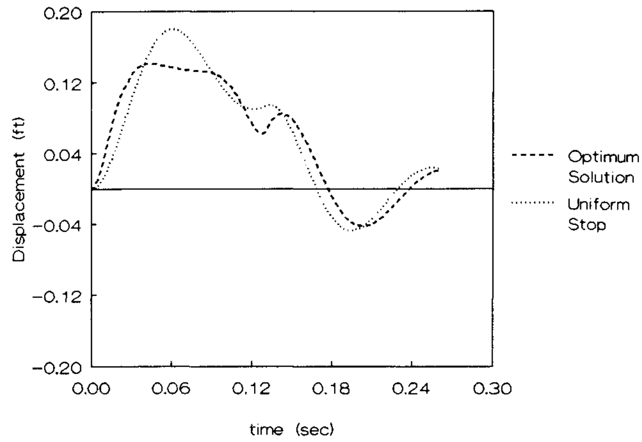

Fig. 4 Spinal model displacement δ for the ejection seat example; conditions as in Fig. 3.

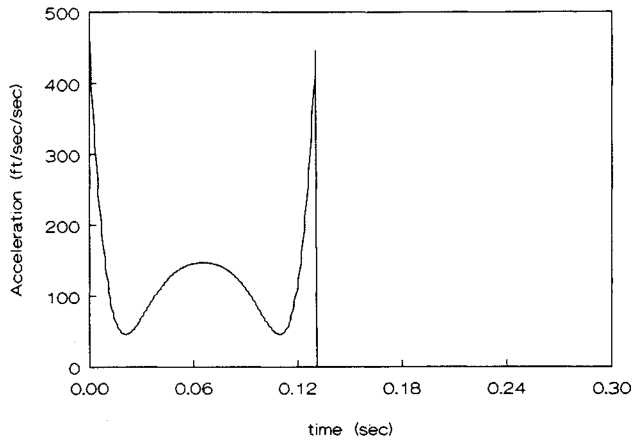

Fig. 3 Optimum acceleration profile for the ejection seat example. The terminal velocity of the seat is constrained to be 44 ft/s and to be attained in 0.130 s.

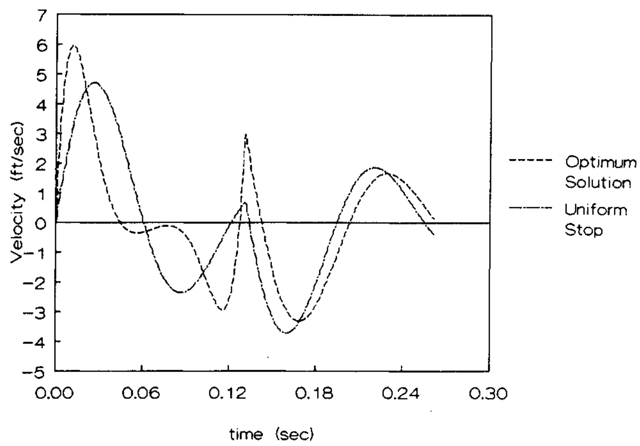

Fig. 5 Spinal model velocity $\dot{\delta}$ for the ejection seat example; conditions as in Fig. 3.

The transversality condition (16) is

$$\xi = \alpha \begin{bmatrix} \omega_n^2 \\ 2\zeta\omega_n \\ 1 \end{bmatrix} \tag{24}$$

where α is a scalar value chosen to satisfy Eq. (8).

Results for the Ejection Seat Model

The example scenario is an ejection from an aircraft with the requirement that the seat reach a velocity of 44 ft/s over the catapult stroke of approximately 34 in. This stroke typically occurs in about 130 ms. These values provide adequate clearance for current high-performance aircraft.

Stech and Payne[9] computed values for ζ and ω_n that were representative of the U.S. Air Force flying population prior to 1969 (mean age 27.9 years, acceleration level 20 g). These values remain in use today for ejection seat specification and will be used in this calculation. The values are

$$\zeta = 0.224$$

$$\omega_n = 52.9 \text{ rad/s}$$

Figure 2 shows, for this same population, spinal injury incidence as a function of DRI.[4]

Spinal Dynamics

For the scenario just described, the optimum ejection seat acceleration profile is shown in Fig. 3. A value of $r = 0.1$ was found to produce achievable and humanly tolerable accelerations. Note the early impulsive acceleration, followed by a short ramp up to a value that is nearly constant until late in the acceleration period when there is a late acceleration pulse that mirrors the initial pulse. Apparently, because of the relationship between the system differential equation (4) and the transversality condition (10), $u^*(t)$ will be symmetrical about $T/2$. (The author was unable to prove this in general; however, in many calculations with different criterion functionals and with the stated relationship between the system equation and transversality condition, the symmetry held. Any disturbance in this relationship destroyed the symmetry.) The initial behavior of u^* is roughly consistent with the calculations of Payne[6] which showed an advantage to an early impulsive loading with a ramp up to a constant acceleration, and the experiments of Hearon et al., which showed an advantage to acceleration preloading. The late acceleration pulse is a finding that suggests a new design consideration.

Spinal model displacement is shown in Fig. 4. Here, the advantage of the optimum acceleration profile is clear. The peak displacement is much lower than that produced by the required uniform (square-wave) acceleration pulse of 339 ft/s^2 lasting 130 ms. From Fig. 2, it can be seen that these DRI values translate to respective injury incidence of less than 0.2% and greater than 2% for the optimum and uniform acceleration cases. This is a significant reduction in injury potential.

Spinal model velocity is shown in Fig. 5. Except for the initial velocity peak and acceleration offset peak, the optimum control maintains lower peak velocities throughout the acceleration and free motion. The time functions show that spinal motion is stilled rather quickly by the optimum control.

Ejection Seat Dynamics

Because the optimum acceleration function is nearly constant for much of the acceleration period, there is little difference between ejection seat dynamics for the two cases, as can be seen in Figs. 6 and 7. Of course, both cases reach a velocity of 44 ft/s in 0.130 s with the optimum acceleration case producing slightly more seat travel early in the catapult stroke.

Physical Attributes of the Model

Although the optimum control significantly reduces the predicted injury potential for the ejection model, the form taken by the optimum control is not particularly intuitive. As stated, the symmetry in u^* apparently derives from the essential equivalence of the problems "accelerate the sled from rest to V_0" or "decelerate the sled from V_0 to rest" with identical initial conditions on the cargo model in both cases; this equivalence is summarized in the particular relationship between the differential equation modeling the cargo and the transversality condition. Some additional insight may be gained by observing the behavior of u^*, x_2^*, and x_3^* as r is varied.

In order that the physical behavior of the model can be observed more easily, a less stiff and less damped cargo model is used in the following example. The defining parameters are

$$\zeta = \frac{1}{2\sqrt{2}}$$

$$\omega_n = \sqrt{50} \text{ rad/s}$$

$$V_0 = 10 \text{ ft/s}$$

$$T = 1 \text{ s}$$

Figure 8 shows the optimum acceleration profiles for this model for three values of r chosen to show transitional behavior as r ranges over two orders of magnitude. As should be expected, as u becomes dominant in the MGI (i.e., r large), the optimum acceleration profile becomes uniform; however, the behavior of u^* as $r \to 0$ is not obvious. For $r \to 0$, it is known that the optimum control will be impulsive (Ref. 11, Chap. 9), and so it is tempting to guess that, in the limit as $r \to 0$, impulses will be located at $t = 0$ and T; this has not been proven.

From Figs. 9 and 10, it can be seen that, for small values of r that allow large excursions of u^*, the system is brought nearly to rest in compression for a considerable span of the applied acceleration, but then allowed to decompress a bit just prior to the late acceleration pulse. This has the effect of reducing the MGI during free motion ($t > T$), although it produces a short lived velocity increase at $t = T$. It is interesting to note that the mass is held nearly stationary by an acceleration that is nearly constant and approximately equal to V_0/T, the amplitude of the uniform acceleration pulse. Since the spinal velocity is small during this period, it is easy to see from Eq. (4) that the corresponding spinal displacement is approximately $V_0/(T\omega_n^2)$, which may be used to estimate a minimum attainable DRI from which a boundary on injury potential may be predicted.

Finally, the choice of $n = 2$ in Eq. (1), which was made for analytical tractability, begs a question about sensitivity of u^*

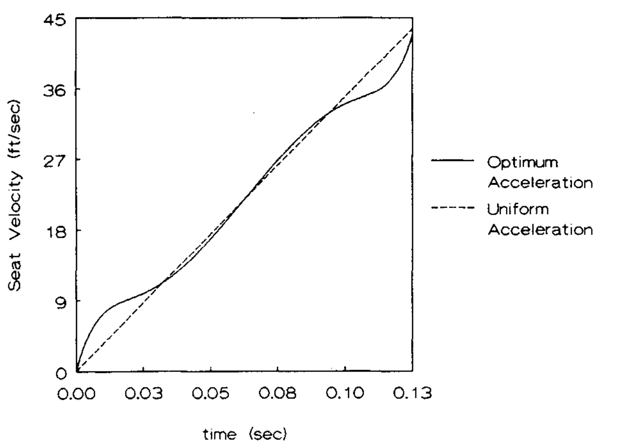

Fig. 6 Ejection seat velocity resulting from the optimum and uniform acceleration profiles for the example. Terminal velocity is 44 ft/s at 0.130 s.

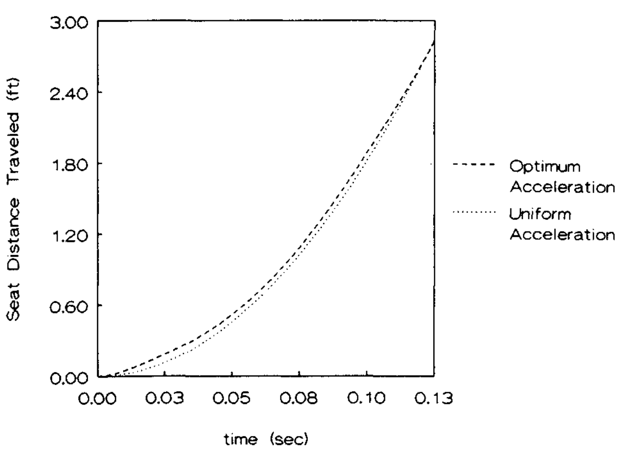

Fig. 7 Distance traveled by the ejection seat for the optimum acceleration profile (Fig. 3) and the uniform acceleration profile. Terminal seat velocity is 44 ft/s in both cases.

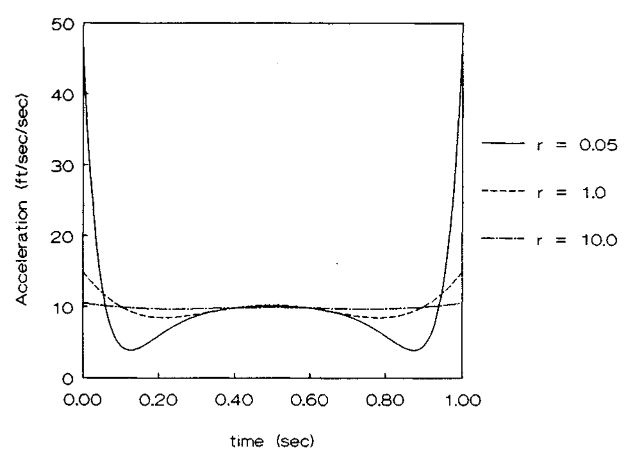

Fig. 8 Optimum acceleration profiles for the illustrative example: $\zeta = 1/(2\sqrt{2})$; $\omega_n = \sqrt{50}$ rad/s; $V_0 = 10$ ft/s; and $T = 1$ s.

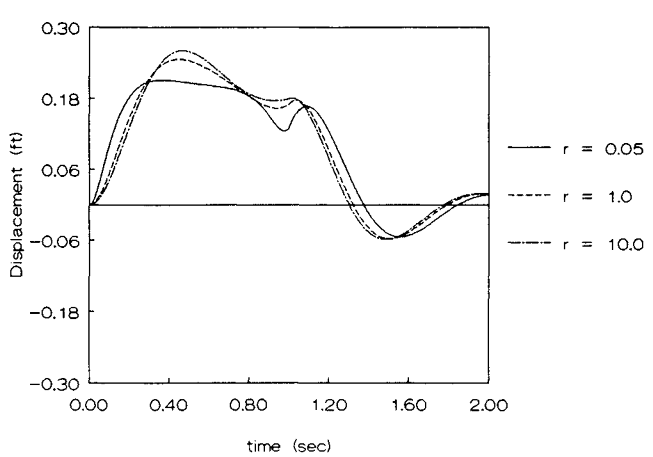

Fig. 9 Displacement profiles for the illustrative example; conditions as in Fig. 8.

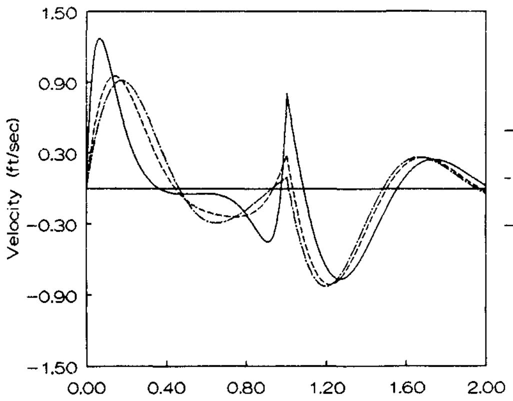

Fig. 10 Velocity profiles for the illustrative example; condition as in Fig. 8.

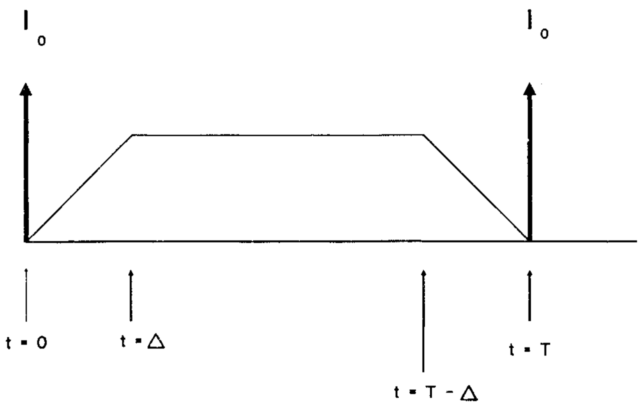

Fig. 11 Idealized acceleration profile.

to a particular value of n. Indeed, u^* may be quite sensitive to this choice since the resulting objective functional may no longer be relevant to the problem at hand (the integrand may no longer be non-negative or even real). The value usually chosen is 2.5, with the acceleration a taken as an absolute value, which derives from head injury data from automobile accidents; the problem addressed here is essentially different and the objective functional (3) has characteristics strongly relevant to the problem addressed herein.

Idealized Acceleration Profile: Design Guidelines

The previous observations lead to the consideration of an acceleration profile that consists of an impulse (Dirac delta function) of magnitude I_0 at $t = 0$, a ramp up to an acceleration plateau of V_0/T at $t = \Delta$, and symmetry about $T/2$. Such an idealized profile is shown in Fig. 11 and may be of some use in design. For this profile, Eq. (8) requires that

$$I_0 = V_0 \frac{\Delta}{2T} \tag{25}$$

The GI may be computed directly by using this profile in Eq. (3) with $r = 0$, and a value of Δ determined that minimizes the index; this calculation is straightforward and is not included here. Figure 12 shows a comparison of displacement profiles using this idealized acceleration profile with those using the optimum and uniform acceleration profiles for the ejection seat model. The optimum value of Δ was found by the technique described above to be 0.03. It is clear that the idealized model provides a reasonable rule of thumb for incorporating in the design process if some attention is given to determining a best value for Δ, and if care is taken to ensure that the amplitude-width characteristics of the impulsive load are within human tolerance.

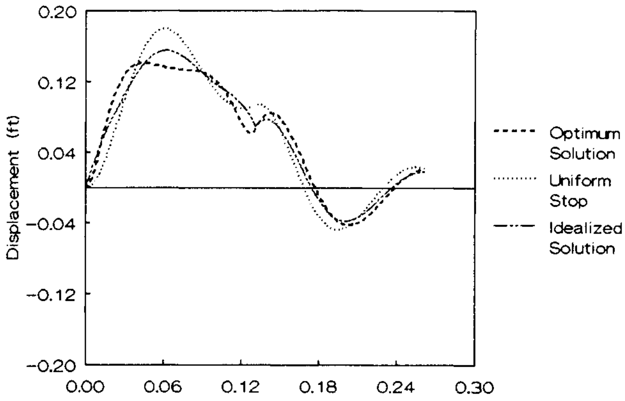

Fig. 12 Comparison of ejection seat model displacement profiles resulting from the optimum acceleration profile, a uniform acceleration pulse, and the idealized acceleration profile; conditions as in Fig. 3.

It is satisfying that the results derived herein are roughly consistent with earlier ad hoc, but educated, approaches to minimizing injury during aircraft ejection, and it is heartening that the optimum control model provides a significant reduction in injury potential for the simple example presented.

The structure of the optimum control model was not dwelled upon, partly because the structure of such linear controls is covered adequately in extant literature, but most important because it would be extremely unlikely that acceleration systems employing rockets could be thrust controlled with enough precision to employ such a feedback loop; the most likely approach would be to contour the catapult rocket burn to meet some specified acceleration profile. Of course, scenarios such as an automobile breaking action may allow for closed-loop control.

Conclusions

The optimum control model presented here is very simple; nevertheless, it has allowed the introduction of new considerations into the design of systems that accelerate vehicles containing human cargo. Although it employs a spinal injury model that fits epidemiologic data quite well, the extensions made herein should be validated empirically. Some experiments should be accomplished using recently introduced high-fidelity dynamic simulators (manikins) to determine if significant gains are to be realized using these concepts.

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